

Introduction to Portfolio Theory

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Summary

The lecture is divided in an introductory macroeconomic and a technical mathematical part. With this the first three chapters present the fundamentals of portfolio theory, like the Bernoulli- resp. the mean-variance principle, the portfolio selection using efficient portfolio lines and the CAPM. While up to now the pure discrete trading models were used one starts with the treatment of time continuous asset models. First the basic pricing models and facts for portfolios are given, like trading strategies and complete markets. Afterwards it is shown that under certain mild assumptions an optimal portfolio selection can be achieved. As decisive tool we give an introduction to optimal control theory in both cases of finite and infinite time horizon for the consumption process. The lecture is completed by a short view in the portfolio theory using derivatives as security instruments.

We need an introductory lecture on mathematical finance for the understanding of this content, since the knowledge on stochastic differential equations is fundamental. Depending how detailed the lecture is presented it would require a course of 3 to 4 hours per week.

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