

# Life Insurance Mathematics (Advanced)

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## Aims

This course provides a rigorous study of advanced topics in life insurance mathematics. It offers the student the theoretical concepts needed by a life insurance actuary.

## Prerequisites

- Operational knowledge of probability theory and statistics.
- Knowledge of basic life insurance theory.

## Contents

The course starts with a brief overview of *basic life insurance theory*: survival distributions and life tables, life insurance, life annuities, net premiums and net premium reserves.

The basic theory is first extended to insurance and annuities on a general status, with benefits and premium payment plans dependent on several lives. *Multiple life functions* depending on joint-life and last survivor statuses are studied, as well as insurance products where the order of death determines the benefits and premium payments.

In basic life insurance theory, individual lives (or statuses) are subject to a single decrement of death. *Multiple decrement models* are developed for situations where a single life is subject to multiple decrements (withdrawal, death, disablement, retirement). A major application of multiple decrement models in actuarial science can be found in the theory of pension plans. We consider basic methods used in calculating the actuarial present value of benefits and contributions for a participant in a pension plan. The multiple decrement model is generalized to the *Markov model for life contingencies* where transitions between different states are possible in both directions.

The main cash outflows for an insurer are the benefits to be paid out upon survival or death of the insured. But an insurer has also cash outflows related to expenditures. *Insurance models including expenses* start from the equivalence principle to establish an actuarial equivalence between gross premiums at the one hand, and benefits and costs at the other hand. Gross reserves, surrender values and zillmerised reserves are considered in detail.

Conventional insurance contracts are contracts where the benefits (upon survival or death) as well as the premiums are known (deterministic) at policy issue. For such contracts, the investment risk is taken by the insurer. We investigate several alterations of the classical life insurance. *Universal life insurance* contracts offer more flexibility compared to conventional contracts in the sense that premium payments are not fixed at policy issue. In *Unit-linked* life insurance contracts the investment risk is (partially) taken by the policyholder.

The concept of *Fair Valuation* of life insurance liabilities is discussed.

*Profit testing* is a technique used for determining expected profits of a given contract (conventional or unit linked). It is used to determine premiums and/or cost structure of a contract (conventional or unit-linked) such that a predetermined profit target is reached.

Attention is also given to *Embedded Value* calculations (calculation of the Present Value of Future Profits and of the Net Asset Value).

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## Exam

Written exam.

## Reading Material

- Bowers, N.L.; Gerber, H.U.; Hickman, J.C.; Jones, D.A.; Nesbitt, C.J., (1997). *Actuarial Mathematics*. The Society of Actuaries, Itasca, Illinois. Second edition.
- De Vijlder, F.E. (1997). *Life Insurance Theory: Actuarial Perspectives*. Kluwer Academic Publishers, Boston, Dordrecht, London, pp. 184.
- Gerber, H.U. (1995). *Life Insurance Mathematics*. Second edition, Springer, Swiss Association of Actuaries, pp. 217
- Hardy, M. (2003). *Investment Guarantees: Modeling and Risk Management for Equity-Linked Life Insurance*. Wiley, pp. 286.
- Möller, T.; Mogens, S. (2007). *Market-Valuation Methods in Life and Pension Insurance*. Cambridge University Press, pp. 279.
- Wolthuis, H. (1994). *Life Insurance Mathematics, the Markovian Model*. Caire Education Series 2, Brussels, pp. 255.