

Pricing Derivatives with MATLAB

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Summary

After the introduction to the Monte Carlo methods and their implementation in MATLAB, these tools will be applied to the solution of special stochastic differential equations, which appear, when exotic options are priced. Next numerical methods for solving parabolic partial differential equations are treated, as the finite difference method or the vertical line method. The implementation in MATLAB finishes the chapter. Because of the special structure of american style options, free boundary value problems and their numerical solutions are considered in the last chapter, which is again closed by the implementation in MATLAB. In every chapter the results are explained in detail, by presenting examples of special derivatives.

The lecture is based on introductory ones in finance mathematics and pricing of derivatives. Depending how detailed the lecture and the programmes are presented it is a course of 3 to 4 hours per week.

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